On-Line Appendix: Real and financial crises in the Keynes-Kalecki structuralist model: An agent-based approach

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Abstract

This on-line supplemental appendix relaxes some of the key assumption of the main text in order to determine the extent to which its principal findings remain in tact. In particular, the number of lending plies is changed from one to two the number of financial agents is increased from 1000 to 1500. Other changes in the basic structure and assumptions of the model are also considered such as breadth versus depth lending search and recontracting, in which financial agent who take part in a loan consortium that falls apart can quickly redirect there lending capacity to other deficit firms. It is seen that despite theses rather significant changes in the modeling structure, there is little substantial change in the conclusions of the main paper.

Keywords: systemic risk; crash; herding; Bayesian learning; endogenous money; preferential attachment; agent-based models.
JEL codes: D58, E37, G01, G12, B16, C00.

1. Introduction

Several key assumptions are modified in this appendix to gauge the sensitivity of the main results of the paper. The first modification is to the number of "plies" to which each deficit firm has access when seeking to obtain financing from surplus firms. Deficit firms have access to finance through local lenders, financial agents that are co-located with the firms. If the forecast of the local lender is positive and the financial agent has sufficient liquidity, the local lender will make a loan to the deficit firm to cover the desired or planned investment of the deficit firm.

If there is more than one local lender associated with the firm and the first local lender has insufficient liquidity to finance the loan request, a second local lender may be asked to complete the loan request. If a loan package can be arranged among local lenders, each with a positive forecast but insufficient liquidity to finance the investment on their own, the loan is made and planned investment of the firm is undertaken.

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If the local lender is unable to cover the loan because of insufficient liquidity, the local lender can call on a link neighbor, a lender directly connected to the local lender by way of an edge of the network. Asking a link neighbor for a loan will only take place if the forecast of the local lender is positive. The latter depends on a weighted average of public and private signals plus a random error term, as discussed in the main paper.

Firms can either be in surplus or deficit depending on the conditions of aggregate demand. Firm identity is not fixed, however, and a firm could be in surplus in one sweep of the model but in deficit in another. The liquidity that the financial agent makes available to the firm is therefore endogenous. A firm that remains in deficit for several sweeps will not make any deposits with its financial agents and eventually all investment will have to be made through its link neighbors. Neighbors linked directly to local lenders are considered "one-ply" deep and if no loan package can be arranged, finance for the deficit firm is denied. In a two-ply system, not only are link neighbors asked to make loans to deficit firms, but also link neighbors of those link neighbors. This, of course, greatly increases the access of any deficit firm to surplus firms. It may, however, reduce the local lending ability of the two-ply lenders to their own firms. The aggregate effect of a system of one- versus two-ply lending is therefore an empirical matter.

The model of the main paper is run under a one-ply lending assumption. With a two-ply system, a binding financial constraint, a preferentially attached network and public signals (weighted by the share of total capital stock of the firms to which the lending financial agents are associated), the number of crashes *falls* from 8.2 as reported in table 6 of the original paper to 7.2 per 1000 runs.[2] This is entirely consistent with expectations. If there are few limitations on borrowing, the tendency for the system to crash is reduced. These data show a clear difference in results for the one- versus two-ply systems but it remains to be seen whether this this difference is statistically significant. This question is addressed below in detail.

A second crucial question is whether the number of financial agents affects the performance of the system. Again, with more sources of finance, one would expect a better performing economy, both in terms of GDP and capacity utilization. Higher levels of utilization should improve forecasts, which in turn would cause the financial system to work in a more accommodating fashion. As seen in detail below, however, the number of crashes per 1000 runs actually *increases*. The reason is not a lack of finance, but rather the run-up in the price of assets as recorded in the model. A crash requires a "build", which in turn requires some strong economic performance leading to that build. An overly accommodating financial system so reduces "sand in the gears" as to raise the possibility, from time to time, of a build that leads to a crash. Conversely, an economy stuck at low levels of capacity utilization may not crash simply because it never had the wherewithal to produce an adequate build.

In general, the regressions of this appendix show that the model still behaves as it does in the main text. GDP and share price are negatively and highly significantly dependent

[2]Money is endogenous for both.

on the number of loans denied. Crash frequency depends on contagion that arises from a process of Bayesian updating of public signals via a private signal. Aggregate demand is redistributed in a way that prevents any firm from exceeding its capacity to produce. In short, nothing in this appendix rises in contradiction to the main findings of the paper but rather provides support for the generality of the principal conclusions therein.

For the purposes of this appendix, an additional 16 simulations of 900 runs each (for a total of 14,400) were performed, each using various settings of the model. The total number of observations in this exercise is 18,907,810. Table 1 summarizes the data. There it is seen that the time step ranges from 1 to 1500, or 30 years, as in the main text. Most runs were completed in this sample. As explained in the text, runs with secular decline that led to utilization of capacity less than 60 percent were halted. If there had been no capacity floors or crashes, the mean time would have been 750 rather than the observed 748.6 The large standard deviation suggests that capacity floors and crashes were distributed throughout the time profile of the runs.

The average price across all the additional runs was 3.488 with a standard deviation of 1.031. The maximum price observed, 11.188, is 7.46 standard deviations above the mean. The table shows 97 total crashes, where a crash requires a build and decline as specified in the text. The weight variable is κ as in the text and is the Bayesian updating parameter that shows the weight on the public signal. A larger κ allows for the possibility of contagion to sweep through the model and push the share price up or down, and possibly cause a crash. The variation in the κ is significant, up to 4.84 standard deviations above the mean and 2.49 below.

Table 1 also shows that the number of loans denied *per sweep* was large, some 150.1. Since roughly half the firms are surplus and other half deficit, this implies that 388 firms need finance. The average number denied is less than half this amount, but the range is large, with the maximum 4.78 standard deviations above the mean and 2.62 below. This shows that the model has a very active financial sector that is more than capable of throwing "sand in the gears" of the real sector.

The mean GDP of these runs was 5.815, similar to that reported in the main text. There was substantial variation, as one would expect in the reported performance of the simulated economy, ranging from 1.82 below to 3.57 standard deviations above the mean. Capacity utilization also varied from its mean of 67 percent, from a low of 25 percent to full capacity utilization.[3] The means of the pubic and private signals were approximately zero, consistent with the main text, but again there was variation. The public signal ranged from 7.16 below to 6.44 standard deviations above the mean, while the private signal range was from 6.55 below to 5.43 above.

[3] Such dramatic collapses in utilization rates were possible but rare, as indicated by the standard deviation.

Table 1: Summary data: 18,907,810 observations (14,400 runs of the model)

Variable	Mean	Std. Dev.	Min	Max
Time	748.6	433.192	1.000	1500
Price	3.488	1.031	1.594	11.188
Crashes	97			
Weight[1]	0.295	0.095	0.057	0.757
Loans denied	150.1	56.91	1.000	422
GDP[2]	5.815	2.972	0.406	16.421
Utilization[3]	67.052	10.215	25.211	100
Public signal	-.0003848	0.082	-0.587	0.528
Private signal	-.0000265	0.005	-0.031	0.025

Source: Authors' computations.
Notes: 1. On private signal. 2. Gross domestic product.
3. Moving average over last 100 runs averaged over all runs.

2. Money not endogenous

Broadly, the simulations of both the paper and this appendix show that if money is not endogenous, there is a collapse in the system.[4] Surplus firms are not able to invest if their financial agents have already lent their surplus to deficit firms. Table 2 shows the results for 1,498,766 observations or 2,236 runs in which money was exogenous. There are also 17,409,044 observations on 11,700 runs.[5] Note that there are far fewer crashes when money is not endogenous. In this case, the system rarely has the requisite build to generate a crash; there is secular stagnation, but this is not the same thing as a collapse in the share price due to financial contagion. Observe that the rate at which loans are denied is elevated, most under the zero-ply assumption. The failure of the financial system to channel resources from surplus to deficit firms is evident.

In the second panel, money is endogenous and the performance of the system is correspondingly improved. Notice the rise in GDP, capacity utilization and the fall in loans denied. Performance is further improved in the penultimate column, in which the system advances from zero to one ply. With two-ply lending, the improvement diminishes but is still present. In general, there are fewer crashes when the financial system works better, that is,

[4] The notion of endogenous versus exogenous money is treated in more detail in Gibson and Setterfield (2015).

[5] Observe that in the case of zero ply, the rate of capacity utilization falls below 60 percent. This simulation is nonetheless included to illustrate the rather dramatic effect of exogenous money on the system.

Table 2: Sensitivity analysis

	Money Endogenous?					
		No			*Yes*	
Ply	*0*	*1*	*2*	*0*	*1*	*2*
Crashes[1]	6.8	0	0	13.4	8.6	7.2
Price	5.344	3.61	3.37	3.38	3.53	3.32
GDP	0.989	2.82	3.04	4.23	6.793	5.64
Loans denied[2]	53.1	32.0	32.0	22.9	19.5	19.1
Capacity utilization[3]	33.9	61.4	61.4	63.9	70.5	72.18
Bayesian prior[4]	0.367	0.292	0.292	0.295	0.296	0.293
Total runs	443	895	898	1800	8100	1800

Source: Authors' computations.
Notes: 1. Crashes per 1000 runs. 2. Percent of total firms.
3. Moving average over last 100 runs averaged over all runs.
4. Average over all runs.

with more plies. This translates into more borrowing opportunities and fewer loans denied.

To what extent are these differences statistically significant? Table 3 shows the results of a series of regressions using the data from all runs in table 1. Crash frequency is the dependent variable and the right-hand-side variables are progressively entered. These regressions are not intended to provide a fully rigorous analysis of the simulations, but rather should be taken as indicative of the relative importance of some of the independent variables included.[6]

The first column is a bivariate regression showing the importance of endogenous money to the crashes. This same relationship survives the addition of multiple regressors as shown in columns 2-4. Observe that the included regressors are in general highly significant. Price, as noted, is positively associated with crashes because a build must precede a crash. The Bayesian prior term speaks to the importance of contagion as a root cause of crisis in the model. Loans denied, also highly significant even with other controls, is the mechanism by which a crash is initiated. Without an incipient rise in loans denied a crash can never materialize. The table shows, generally, that not only are the causal mechanisms discussed above observable in the output data, but are also statistically significant.

[6]GDP and capacity utilization are not included, for example, since the correlation matrix (not shown) indicates a close relationship to loans denied.

Table 3: Regression results

	(1) Crashes[1]	(2) Crashes[1]	(3) Crashes[1]	(4) Crashes[1]
Endog. money	3.398e-06**	5.150e-06***	4.936e-06***	2.220e-05***
	(1.283e-06)	(1.354e-06)	(1.348e-06)	(4.292e-06)
Price		1.102e-05***	1.101e-05***	1.117e-05***
		(1.315e-06)	(1.314e-06)	(1.326e-06)
Bayesian prior			2.882e-05***	2.984e-05***
			(5.676e-06)	(5.748e-06)
Loans denied				9.038e-08***
				(2.159e-08)
Constant	2.002e-06	-3.804e-05***	-4.633e-05***	-7.663e-05***
	(1.156e-06)	(4.789e-06)	(5.671e-06)	(1.035e-05)
R^2	0.000	0.000	0.000	0.000
Observations	1.89e+07	1.89e+07	1.89e+07	1.89e+07
F-stat	7.016	35.198	24.163	18.826

Standard errors in parentheses. *** p < 0.01, ** p< 0.05, * p< 0.1
Notes: 1. Crashes per 1000 runs of 1,500 weeks (30 years).
Source: Author's computations.

3. Strengthening the financial sector: multiple plies and additional financial agents

The most telling regressions appear in table 4 in which the presence of multiple plies enter the regressions in table 3 as additional regressors. Observe first the rather dramatic lack of significance of the one-ply regressor, keeping in mind that the data set includes runs with a zero-ply assumption, that is with no linked lenders whatsoever. The one-ply coefficient has a positive sign, but fails to reach significance even at the ten percent level. This captures numerically the dilemma posed above: when a lender agrees to a linked loan, its ability to service its local borrower is diminished. Even two-ply is only weakly related with at most a 5 percent significance level in the last column. Again, there are opposing forces at work, as the weakness of the regression results confirm.

The effect of adding additional financial agents is similarly weak, with clear explanation. More financial agents cannot *create* more surplus but only redistribute more effectively what has already been produced. If the network structure blocks the distribution, investment in the next period will fall. There is now a consortium of forces at play, opposing one another and creating a less than clear cut regression result.

4. Coding options: Recontracting and breadth v. depth searches

The previous section addressed the fundamental issue of the structure of the financial system. Inevitably, however, some coding choices have to be made in the flow of logic of the program that could potentially affect the qualitative results. This brief section is only suggestive of two such options among a host of others. Fortunately neither has much impact on the qualitative conclusions of the main paper and it will be left to further research to explore various other coding paths within the model.

There is a broad reason however that one would not expect these issue to impinge: the paper deals with the degree to which the financial system aids in the process of accumulation of capital. The spectrum here is continuous from more to less and then, of course, much less when flow of finance is essentially completely blocked. Small variations in the exact location of the needle on this spectrum would not be expected to make a great difference. The results reported in 5 show that this is indeed the case.

4.1. Breadth v. depth lending searches

There are two ways the search for finance can be undertaken. A deficit firm can ask local lenders for finance and if they cannot obtain enough from one, they could ask another local lender to join to complete the financing package. The other option is for a local lender to ask one of its link neighbors (one- or two-ply) first rather than going to other local lenders. The effect of this search by depth rather than breadth, at first blush, may seem inconsequential; but again, the depth search may be more likely to produce additional finance, and therefore fewer crashes, than the breadth search. To see this, consider a firm that is perennially in deficit. Over time, its associated financial agents, the local lenders, will not receive any financial surplus, so all of the lending ability they have will be from link neighbors. This is

Table 4: Regression results

	(1) Crashes[1]	(2) Crashes[1]	(3) Crashes[1]	(4) Crashes[1]
Endogens money	2.225e-05***	2.367e-05***	2.773e-05***	3.058e-05***
	(4.296e-06)	(4.487e-06)	(4.542e-06)	(4.859e-06)
Price	1.116e-05***	1.121e-05***	1.092e-05***	1.113e-05***
	(1.326e-06)	(1.328e-06)	(1.302e-06)	(1.317e-06)
Bayesian prior	2.984e-05***	3.012e-05***	3.036e-05***	3.052e-05***
	(5.748e-06)	(5.762e-06)	(5.795e-06)	(5.804e-06)
Loans denied	9.123e-08***	1.094e-07***	1.345e-07***	1.508e-07***
	(2.208e-08)	(2.527e-08)	(2.618e-08)	(2.789e-08)
One-ply	1.864e-07	2.561e-06	2.407e-06	2.772e-06
	(1.147e-06)	(1.599e-06)	(1.589e-06)	(1.597e-06)
Two-ply		4.397e-06*	5.180e-06*	5.669e-06**
		(2.079e-06)	(2.124e-06)	(2.139e-06)
Agents[2]			4.281e-06*	4.559e-06*
			(2.143e-06)	(2.134e-06)
Time				-4.022e-09**
				(1.358e-09)
Constant	-7.691e-05***	-8.335e-05***	-9.056e-05***	-9.372e-05***
	(1.046e-05)	(1.145e-05)	(1.203e-05)	(1.233e-05)
R^2	0.000	0.000	0.000	0.000
Observations	1.89e+07	1.89e+07	1.89e+07	1.89e+07
F-stat	15.064	12.706	10.902	.

Standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$
Notes: 1. Crashes per 1000 runs of 1,500 weeks (30 years). 2. A 50 percent increase in the number of financial agents.
Source: Author's computations.

even more true if there is only one financial agent serving the firm. However, when a local lender first goes to its link neighbor, finance is more likely to be available given that the link neighbor will be servicing a surplus firm with a probability approximately given by the proportion of surplus firms.

Does this in fact matter with respect to the crash propensity of the grid as a whole? One might argue not; if the local lenders are searched first and they can meet the demand for loans then the deficit firm can invest. If one of the local lenders can meet the demand for loans using a link neighbor, then there is, in fact, no difference in the total amount of finance forthcoming. Since there is no time limit on how long the search proceeds, it seems that breath v. depth search will produce the same number of crashes. Table 5 shows that this is not strictly the case; there is some scant evidence that depth first search does indeed reduce crash frequency. Despite the absurdly high number of degrees of freedom in the regression, the depth first dummy only achieves marginal significance, however.

4.2. Recontracting

The paper does not allow firms to switch from deficit to surplus during a single sweep of the model. If a deficit firms fails to arrange financing, it could be argued that the firm has converted from a deficit firm to a surplus firm since the level of investment is set at zero. At that instant the total amount of surplus would, in principle, increase. The savings of the deficit firm could then be used to finance other deficit firms. Once denied the opportunity to invest, the deficit firm could deposit its savings with its financial agent, which then could respond to a request for financing by one of its link neighbors. In principal this would increase the amount of finance available on the grid, raise investment and output, thereby altering the propensity of the system to crash.

A modified version of the model was constructed to see if the assumption of a single loan application per sweep made any difference in the crash propensity. Table 5 provides evidence that, in fact, this change is not important. Columns 3 and 4 show that the dummy variable in the regression indicating the possibility of recontracting is not statistically significant.

5. Conclusions

This appendix challenges some of the basic assumptions of the main text. In fact, however, the qualitative conclusions of the text remain remarkably unscathed. This is not by careful choice of experiment; every experiment conducted has been faithfully reported here. Further investigations of the sensitivity of the conclusions are possible, of course, but are left to future research. The results presented in this appendix suggest that they are unlikely to reveal fundamental challenges to the conclusions reached in the main paper.

Gibson, B. and M. Setterfield (2015). Intermediation, money creation, and keynesian macrodynamics in multi-agent systems. Technical report, University of Vermont.

Table 5: Regression results

	(1) Crashes[1]	(2) Crashes[1]	(3) Crashes[1]	(4) Crashes[1]
Endogens money	2.808e-05***	2.902e-05***	3.107e-05***	3.188e-05***
	(4.590e-06)	(4.697e-06)	(4.907e-06)	(5.020e-06)
Price	1.092e-05***	1.093e-05***	1.112e-05***	1.113e-05***
	(1.302e-06)	(1.303e-06)	(1.316e-06)	(1.317e-06)
Bayesian prior	3.035e-05***	3.036e-05***	3.058e-05***	3.057e-05***
	(5.795e-06)	(5.795e-06)	(5.805e-06)	(5.805e-06)
Loans denied	1.360e-07***	1.408e-07***	1.537e-07***	1.576e-07***
	(2.636e-08)	(2.686e-08)	(2.808e-08)	(2.860e-08)
One-ply	3.039e-06	4.120e-06*	2.676e-06	3.799e-06*
	(1.696e-06)	(1.836e-06)	(1.683e-06)	(1.850e-06)
Two-ply	5.934e-06**	6.420e-06**	6.599e-06**	6.985e-06**
	(2.301e-06)	(2.344e-06)	(2.322e-06)	(2.361e-06)
Weighted network	1.484e-06	2.255e-06	1.688e-06	2.313e-06
	(1.237e-06)	(1.340e-06)	(1.253e-06)	(1.342e-06)
Agents[2]	4.436e-06*	3.923e-06	5.651e-06**	4.980e-06*
	(2.151e-06)	(2.162e-06)	(2.123e-06)	(2.131e-06)
Depth search		-4.893e-06**		-4.243e-06**
		(1.518e-06)		(1.528e-06)
Recontracting			3.111e-06	2.347e-06
			(1.646e-06)	(1.676e-06)
Time			-4.041e-09**	-4.076e-09**
			(1.357e-09)	(1.360e-09)
Constant	-9.260e-05***	-9.505e-05***	-9.633e-05***	-9.836e-05***
	(1.243e-05)	(1.272e-05)	(1.275e-05)	(1.303e-05)
R^2	0.000	0.000	0.000	0.000
Observations	1.89e+07	1.89e+07	1.89e+07	1.89e+07
F-stat	9.561	8.639	.	.

Standard errors in parentheses. *** p < 0.01, ** p< 0.05, * p< 0.1
Notes: 1. Crashes per 1000 runs of 1,500 weeks (30 years). 2. A 50 percent increase in the number of financial agents.
Source: Author's computations.